

# The Friedman Test with MS Excel

## In 3 Simple Steps

The screenshot displays an Excel spreadsheet with a data table and a dialog box for performing a Friedman Test.

**Data Table:**

| Person | Monday | Tuesday | Wednesday | Thursday |
|--------|--------|---------|-----------|----------|
| 1      | 2.8    | 3.6     | 1.4       | 2.1      |
| 2      | 5.9    | 1.7     | 0.9       | 2.2      |
| 3      | 3.3    | 5.1     | 1.1       | 0.9      |
| 4      | 4.4    | 2.2     | 3.2       | 1.1      |

**Friedman Test Dialog Box:**

- Worksheets: Friedman(Input), Friedman(Output), Sheet1, Sheet2
- Input Range: 'put')!\$B\$1:\$E\$10
- Grouped By: ☒ Columns, ☐ Rows
- ☒ Labels in First Row
- Significance Level (alpha): 0.05
- Buttons: Execute, Exit
- Footer: For comments, please contact gwet@advancedanalyticsllc.com

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# Preface

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The purpose of this document is to show you a few simple steps for implementing the Friedman test using Excel<sup>1</sup>. The proposed solution is based on a user-friendly Excel macro program called **Friedman-Test.xlsm**, which requires no installation at all and does all the work for you. It is provided in the form of a stand-alone Excel workbook that you may use to input your data, perform your analysis, and save your results independently of the current configuration of MS Office. The only requirement is to have MS Office installed under the Windows operating system.

Section 1 provides a detailed description of the Excel solution proposed for implementing the Friedman test along with the screenshots, and assumes that you already possess a working knowledge of this statistical procedure. In case you need a technical description of the statistical techniques that underly the Friedman test, you will find it in section 2. In addition to describing the Friedman test, section 2 presents the advantage of showing the specific equations that were programmed in the Excel VBA<sup>2</sup> program for interested readers. The implementation of the Friedman test proposed here is practical, intuitive, and easy to use. You use it by following specific and detailed instructions provided in this document. This is the ideal solution for students and researchers using the Friedman test occasionally.

Note that the purchased version of this document is almost identical to the free version with one difference. The purchased version contains the following two items not found in the free version:

- ▶ The actual **Friedman-Test.xlsm** workbook for performing the calculations,
- ▶ A complimentary PDF file of chapter 10 of my Statistics book<sup>3</sup> on the

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<sup>1</sup>The proposed solution has been successfully tested with Excel 2007 and Excel 2010. I expect it to work smoothly with Excel 2003, and perhaps with some older versions of Excel as well

<sup>2</sup>VBA stands for Visual Basic for Applications, and is the programming language used to automate various Excel tasks

<sup>3</sup>Gwet, K.L. (2011), *The Practical Guide to Statistics: Basic Concepts, Methods and Meaning. Application With MS Excel, R, and OpenOffice Calc*

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Analysis of Variance (ANOVA) and related tests such as the Kruskal-Wallis and Friedman tests, or the ANOVA for dependent and independent samples.

The 2 attachments to the purchased version of this document can be retrieved by clicking on the paper clip picture at the bottom left side of the PDF file you are reading. You may need to save them in a directory of your choice before using them.

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# The Friedman Test Implementation with MS Excel

This section shows you step by step how to go from your input dataset to the the Friedman test results, and their interpretation using MS Excel. The proposed solution is compatible with the 2010, 2007, and 2003 versions of Excel. The introductory section 1.1 provides a high-level overview of the proposed Excel solution, while section 1.2 describes the output that you will obtain after conducting the test. In section 1.3, you will see the specific steps to follow for solving the problem.

## 1.1 Introduction

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Excel itself does not implement the Friedman test. Even the “Data Analysis ToolPak” add-in, which is the collection of data analysis tools that comes with Excel does not offer a module for conducting the Friedman test. That is why I developed the `Friedman-Test.xlsm` macro program as a convenient tool for Excel users wanting to implement the Friedman test. Even non users of Excel, who have access to Excel could use this proposed method with no anticipated difficulty.

To illustrate how the `Friedman-Test.xlsm` macro program works, I will use the data shown in Table 1.1. This table shows return rate data in a men’s clothing store by salesperson and day of the week. The problem of interest here is to test if the return rate depends on the day of the week, using the daily return rates associated with the same group of 9 salespersons. Table 1.1 provides a limited amount of information on the daily return rates based a few days and a few salespersons. This will result in a measure of return rate that is subject to sampling error. That is why you must use a statistical test that

accounts for this error, before you can determine whether the return rate is affected by the day of the week or not.

To compare these 4 days of the week with respect to their return rates, I decided to use the Friedman test rather than the traditional ANOVA due to the fact that the same sample of salespersons is used for all 4 days, and that the conditions for using ANOVA for dependent samples were not met<sup>1</sup>.

**Table 1.1 :** Return Rates of clothing in a men's clothing store by salesperson and day of the week

| Day       | Salesperson |     |     |     |     |     |     |     |     |
|-----------|-------------|-----|-----|-----|-----|-----|-----|-----|-----|
|           | 1           | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
| Monday    | 2.8         | 5.9 | 3.3 | 4.4 | 1.7 | 3.8 | 6.6 | 3.1 | 0.0 |
| Tuesday   | 3.6         | 1.7 | 5.1 | 2.2 | 2.1 | 4.1 | 4.7 | 2.7 | 1.3 |
| Wednesday | 1.4         | 0.9 | 1.1 | 3.2 | 0.8 | 1.5 | 2.8 | 1.4 | 0.5 |
| Thursday  | 2.0         | 2.2 | 0.9 | 1.1 | 0.5 | 1.5 | 1.4 | 3.5 | 1.2 |

## 1.2 Output of the Friedman-Test.xlsm Macro

The Friedman-Test.xlsm macro will produce the results shown in Figures 1.1, 1.2, and 1.3 on the same Friedman(Output) worksheet. The first output table of Figure 1.1 displays basic summary statistics computed from the raw data of Table 1.1. The second output table of Figure 1.1 on the other hand, displays the summary statistics based the ranks<sup>2</sup>, as well as the final test results highlighted in yellow.

- ▷ The  $F_r$  statistic is the Friedman test statistic without tie correction (see sub-section 2.2 of section 2 for more details about the correction factor, its purpose and its use)
- ▷ The tie-corrected  $F_r$  statistic is what is typically used in most professional statistical packages.
- ▷ The significance level ( $\alpha$ ) is a value you must supply.

<sup>1</sup>Actually some of the validity conditions associated with ANOVA for dependent samples are quite difficult to verify. See Gwet, K. (2011) for a discussion of these conditions

<sup>2</sup>Note that the ranks are calculated separately for each salesperson (i.e. the subject, the factor under investigation being the day), and the summary statistics produced for each day (across salespersons).

- ▷ The critical value is the threshold that must be exceeded by the test statistic for the null hypothesis of equality in return rates across days to be rejected. *For this particular example, the adjusted and unadjusted test statistics both exceed the critical value, leading to the rejection of the null hypothesis*
- ▷ **The P-value.** In this example, the p-value equals 1.96E-02, which is the scientific notation for  $1.96/10^2 = 1.96/100 = 0.0196$ . This quantity is smaller than the significance level, which will also cause the rejection of the null hypothesis. **Friedman-Test.xlsm** calculates the p-value based on the tie-corrected test statistic.

| A2 |                                                        |                                            |          |           |          |   |
|----|--------------------------------------------------------|--------------------------------------------|----------|-----------|----------|---|
|    | A                                                      | B                                          | C        | D         | E        | F |
| 1  | Friedman Test - Executed on: 5/30/2011, at: 1:07:45 PM |                                            |          |           |          |   |
| 2  |                                                        |                                            |          |           |          |   |
| 3  |                                                        | Summary Statistics on Input Data           |          |           |          |   |
| 4  |                                                        | Monday                                     | Tuesday  | Wednesday | Thursday |   |
| 5  | Count of Observations:                                 | 9                                          | 9        | 9         | 9        |   |
| 6  | Sample Sum:                                            | 31.6                                       | 27.5     | 13.6      | 14.3     |   |
| 7  | Sample Mean:                                           | 3.511111                                   | 3.055556 | 1.511111  | 1.588889 |   |
| 8  | Sample Standard Deviation:                             | 2.013979                                   | 1.367581 | 0.908907  | 0.886629 |   |
| 9  |                                                        |                                            |          |           |          |   |
| 10 |                                                        | Summary Statistics on Ranks & Test Results |          |           |          |   |
| 11 |                                                        | Monday                                     | Tuesday  | Wednesday | Thursday |   |
| 12 | Count of Observations:                                 | 9                                          | 9        | 9         | 9        |   |
| 13 | Rank Sum:                                              | 28                                         | 29       | 15.5      | 17.5     |   |
| 14 | Rank Average:                                          | 3.111111                                   | 3.222222 | 1.722222  | 1.944444 |   |
| 15 | Number of Tie Series:                                  | 35                                         |          |           |          |   |
| 16 | Average Number of Ties by Series:                      | 1.028571                                   |          |           |          |   |
| 17 | Fr-statistic (Unadjusted):                             | 9.766668                                   |          |           |          |   |
| 18 | Fr-statistic (tie-Corrected):                          | 9.876407                                   |          |           |          |   |
| 19 | Significance Level:                                    | 0.05                                       |          |           |          |   |
| 20 | Critical Value:                                        | 7.814728                                   |          |           |          |   |
| 21 | P-value:                                               | 1.96E-02                                   |          |           |          |   |
| 22 |                                                        |                                            |          |           |          |   |

**Figure 1.1.** Results of the Friedman Test applied to Table 1.1 Data

The first output table on Figure 1.2 summarizes the pairwise comparisons, also known as the post-hoc analysis. This analysis is conducted when the null hypothesis of equality of means is rejected. Its objective is to identify the specific pair or pairs of factors with a difference in rank sums that is statistically significant, and which may have caused the rejection of the global

null hypothesis<sup>3</sup>. This output table contains a threshold column showing the number that must be exceeded by the rank sum difference before it is deemed statistically significant.

The first 2 columns list the 2 factors being compared. The third column contains the absolute value of the difference in rank sums between the two factors, while the fourth column shows the threshold to be exceeded before the absolute rank sum difference is deemed statistically significant. The last column reports the statistical significance of the respective pairwise comparisons.

|    | A | B | C | D | E | F | G |
|----|---|---|---|---|---|---|---|
| 22 |   |   |   |   |   |   |   |
| 23 |   |   |   |   |   |   |   |
| 24 |   |   |   |   |   |   |   |
| 25 |   |   |   |   |   |   |   |
| 26 |   |   |   |   |   |   |   |
| 27 |   |   |   |   |   |   |   |
| 28 |   |   |   |   |   |   |   |
| 29 |   |   |   |   |   |   |   |
| 30 |   |   |   |   |   |   |   |
| 31 |   |   |   |   |   |   |   |
| 32 |   |   |   |   |   |   |   |
| 33 |   |   |   |   |   |   |   |
| 34 |   |   |   |   |   |   |   |
| 35 |   |   |   |   |   |   |   |
| 36 |   |   |   |   |   |   |   |
| 37 |   |   |   |   |   |   |   |
| 38 |   |   |   |   |   |   |   |
| 39 |   |   |   |   |   |   |   |
| 40 |   |   |   |   |   |   |   |
| 41 |   |   |   |   |   |   |   |
| 42 |   |   |   |   |   |   |   |

| Pairwise Comparisons |           |           |           |             |
|----------------------|-----------|-----------|-----------|-------------|
| Factor 1             | Factor 2  | Abs Rank  |           | Statistical |
|                      |           | Sum Diff. | Threshold |             |
| Monday               | Tuesday   | 1         | 14.45033  | no          |
| Monday               | Wednesday | 12.5      | 14.45033  | no          |
| Monday               | Thursday  | 10.5      | 14.45033  | no          |
| Tuesday              | Wednesday | 13.5      | 14.45033  | no          |
| Tuesday              | Thursday  | 11.5      | 14.45033  | no          |
| Wednesday            | Thursday  | 2         | 14.45033  | no          |

| Nb. of pairwise comparisons | Thresholds |          |
|-----------------------------|------------|----------|
|                             | 2-sided    | 1-sided  |
| 1                           | 10.73517   | 9.009234 |
| 2                           | 12.27667   | 10.73517 |
| 3                           | 13.11237   | 11.65578 |
| 4                           | 13.6805    | 12.27667 |
| 5                           | 14.1084    | 12.74193 |
| 6                           | 14.45033   | 13.11237 |

**Figure 1.2.** Ranks Associated with Table 1.1 Data

The threshold column of the “Pairwise Comparisons” table is based on a maximum number of 6 pairwise comparisons. If you want to use fewer comparisons, you will need different thresholds that are supplied in the second and last output table of Figure 1.2. For example, if all you want is compare Monday and Wednesday to determine if the associated return rates are different (i.e. a 2-sided test), you use 10.73517 as threshold. To see if the Monday rate exceeds the Wednesday’s (1-sided test), then you need to compute the raw rank sum

<sup>3</sup>Sub-section 2.3 of section 2 provides a mathematical description of the pairwise comparison for interested readers

difference (Monday-Wednesday = 28-15.5) from Figure 1.1, and compare it to the 1-sided threshold 9.009234. If the difference exceeds that threshold, then it will considered statistically significant.

Figure 1.3 shows the ranks associated with the 4 days (Monday, Tuesday, Wednesday, and Thursday) in columns I through M. The **Friedman-Test.xlsm** macro displays these ranks for you to see the data that went into the calculation of the test statistic  $F_r$ .

| Ranks Associated with Input Data, by Factor |         |           |          |
|---------------------------------------------|---------|-----------|----------|
| Monday                                      | Tuesday | Wednesday | Thursday |
| 3                                           | 4       | 1         | 2        |
| 4                                           | 2       | 1         | 3        |
| 3                                           | 4       | 2         | 1        |
| 4                                           | 2       | 3         | 1        |
| 3                                           | 4       | 2         | 1        |
| 3                                           | 4       | 1.5       | 1.5      |
| 4                                           | 3       | 2         | 1        |
| 3                                           | 2       | 1         | 4        |
| 1                                           | 4       | 2         | 3        |

**Figure 1.3.** Ranks Associated with Table 1.1 Data

### 1.3 Using the Friedman-Test.xlsm Macro

◆ You need to open the Friedman-Test.xlsm Excel workbook attached to this PDF file. This is accomplished by clicking on the paper clip picture at the bottom left side of the PDF file you are reading as shown in Figure 1.4. For the first time that you open this workbook, you may get a security warning message notifying you that some active content has been disabled. In this case, you should Enable Content.

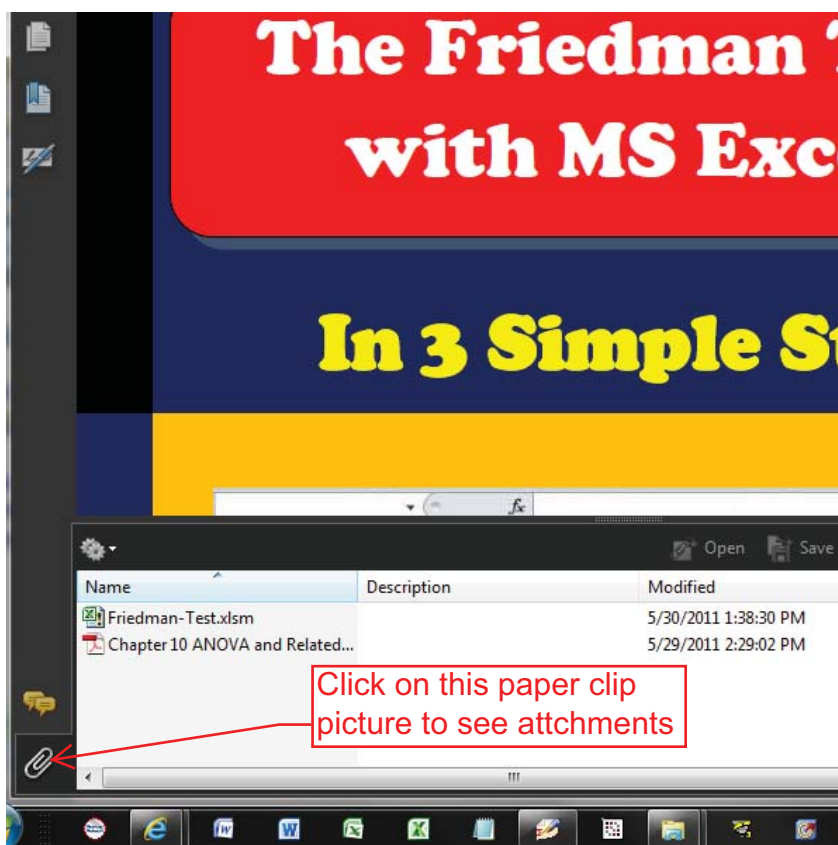


Figure 1.4. Opening the Friedman-Test.xlsm Workbook

◆ Once you open the workbook, you will see 4 worksheets named Friedman(Input), Friedman(Output), Sheet1, and Sheet2. **Never change the names of the first 2 worksheets Friedman(Input), and Friedman(Output)** as this will cause the program to stop working. However you can modify the names Sheet1 and Sheet2 or even add more worksheets as you like. The program will

allow you to select the particular worksheet you like to work with after you click on the “Friedman Test” gray button of Figure 1.5

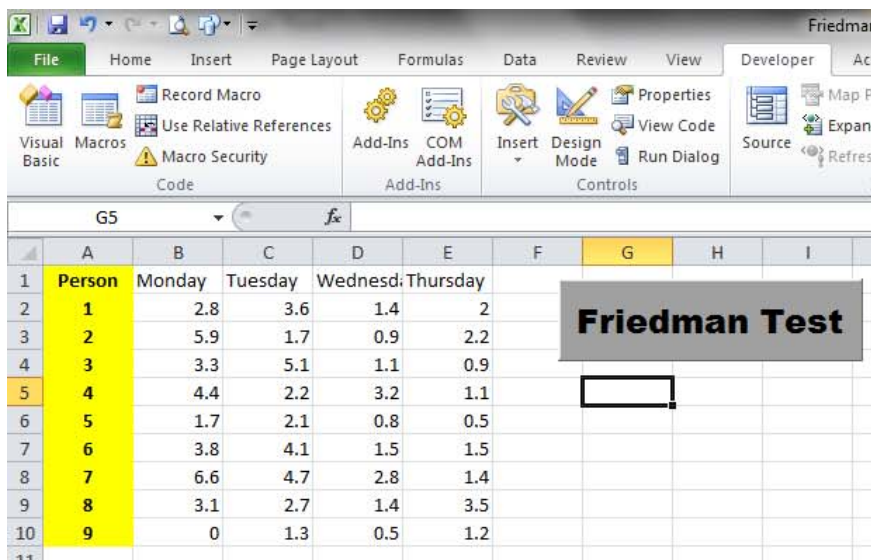


Figure 1.5. Launching the Friedman-Test.xlsm Excel Macro

◆ To conduct the Friedman test, follow the instructions in the next 3 simple steps:

### 1 Launching the Friedman-Test.xlsm Excel Macro

Populate the K-W(Input) worksheet with the data that you want to analyze as shown in Figure 1.5. Then click the **Friedman Test** gray button to launch the program. *The program expects 3 columns of data or more. You may use any worksheet in the Friedman-Test.xlsm workbook to capture your data, except Friedman(Output).*

### 2 Fill out the Dialog Form

Launching the program will display the dialog form of Figure 1.6. (1) Select the worksheet containing your input data from the **Worksheets** list box control. (2) Click inside the “Input Range” RefEdit control with the computer mouse. (3) Using the mouse, select all the data to be analyzed, including column labels as shown in Figure 1.6. (4) Select the “Columns”

radio button if your data is organized columnwise as in Figure 1.6, or select the “Rows” radio button otherwise. (5) Select the “Labels in First Row” checkbox if the first row in the selected range contains the labels. If the first row contains numeric values to be analyzed, then leave that box unchecked. (6) Specify the significance level of the test, if you have one. Otherwise, a significance level of 0.05 will be considered by default.

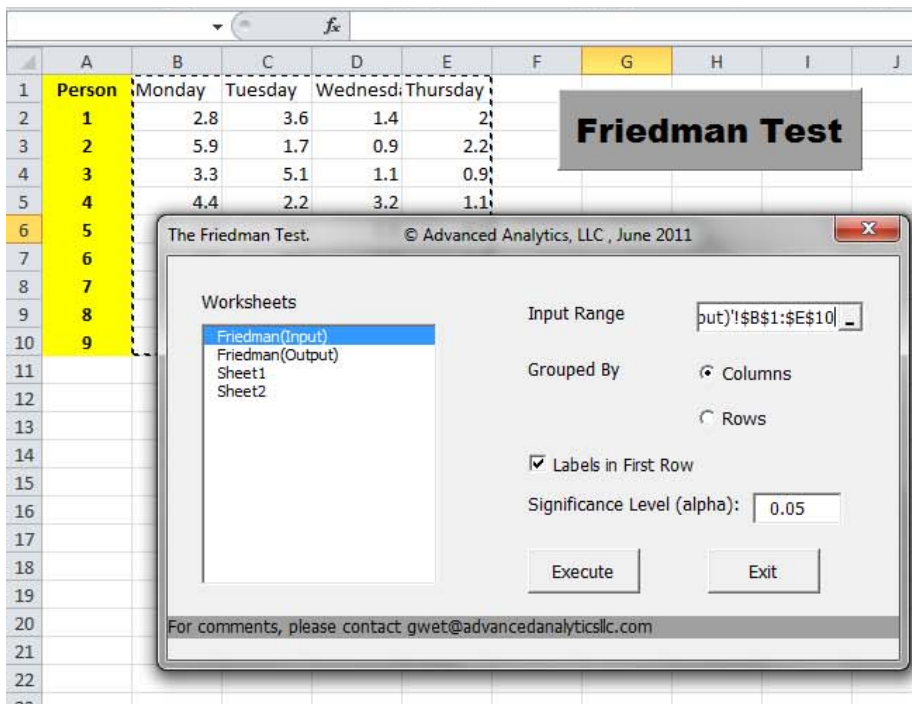


Figure 1.6. Completing the Friedman Dialog Form

### 3 Execute the Macro and Interpret the Results

After filling out the dialog form in step 2, you must click of the **Execute** button, and look at the results in the **K-W(Output)** worksheet. The results will be similar to what is displayed in Figures 1.1, 1.2, and 1.3.

# The Friedman Test Procedure

The Friedman test is typically used for comparing different values of a population mean (or median) evaluated under different conditions, when the number of conditions is 3 or more (these conditions are often seen as different levels of the study factor). The experiment consists of taking measurements from the same sample of subjects on 3 occasions or more under different conditions. For example, you may want to evaluate the impact of surface type (e.g. cement, clay, grass) on the performance of racing horses. You could then select a sample of horses and test that same group on all 3 surfaces. The horses are the sample subjects, while cement, clay, and grass are 3 levels of the surface type factor being investigated. Since the data samples are dependent (because based on the same subjects), neither ANOVA for independent samples nor the Kruskal-Wallis test can be used. ANOVA for dependent samples might not be applicable either if you are concerned about some of its many validity conditions not being satisfied. The only option left will be the Friedman test.

For the sake of comparing 2 population means based on 2 dependent samples, the Wilcoxon matched-pairs signed-rank test should be used. The Friedman test discussed here is an extension of the Wilcoxon test to 3 dependent samples or more. This statistical test, originally proposed Friedman (1937), has been given several names in the literature. Some authors refer to it as the “Friedman Two-Way Analysis of Variance by Ranks” due to the fact that it is based on ranks rather than on the original raw data, and that the *subject* and the *measurement period* are two factors (hence two-way) that affect the magnitude of the measurements. Others have called this technique, “Friedman’s test for randomized block experiment.” This terminology is borrowed from more advanced concepts in the study of ANOVA techniques that are beyond the scope of this book. This test is also sometimes called the “Repeated Measures ANOVA.”

## 2.1 Friedman Test Statistic

The Friedman test should normally be seen as a test of the equality of population medians. However, if the underlying probability distributions are symmetric, then testing population medians amounts to testing population means. Even if the symmetric nature of the underlying distribution is unknown, the test remains valid but only for testing population medians.

The primary goal in comparing three populations or more is to prove that they are different. Therefore, you will naturally want to protect yourself against the possibility of rejecting the hypothesis of homogeneity among the populations when in reality they are all the same. Consequently, if  $k$  dependent populations (or one population observed on  $k$  occasions) are being analyzed, the test hypothesis<sup>1</sup> will be defined as follows:

$$H_0: \mu_1 = \mu_2 = \cdots \mu_k, \quad (2.1)$$

where the symbol  $\mu$  represents the mean or the median depending on whether the underlying distribution is symmetric or not. The alternative hypothesis is defined as,

$$H_a: \text{Not all medians } \{\mu_1, \cdots, \mu_k\} \text{ are equal}^2. \quad (2.2)$$

### Test Statistic

I am assuming here that you have collected  $k$  observations for each of the  $n$  subjects (or experimental conditions) in the study. In Table 2.1 for example, you would have  $k = 4$  and  $n = 9$ . The Friedman's test statistic, which I will denote<sup>3</sup> by  $F_r$  is obtained as follows:

- (a) Rank all  $k$  observations in ascending order<sup>4</sup> from 1 to  $k$  separately for each of the  $n$  subjects (or experimental conditions). If there are ties, then the average of all ranks in a tie series is assigned to each score. *Let  $r_{ij}$  be the rank associated with subject  $i$  and factor  $j$ . That is  $r_{ij}$  may take any value from 1 to  $k$ .*

<sup>1</sup>This will be the "null" hypothesis in Fisher's terminology for P-values.

<sup>2</sup>Note that this alternative hypothesis should not be formulated as  $H_a: \mu_1 \neq \mu_2 \neq \cdots \neq \mu_k$ . This formulation is wrong because the null hypothesis must be rejected even if some means are equal.

<sup>3</sup>This notation is formed with letter F for Friedman and  $r$  to indicate that the statistic is based on ranks, and not on raw measurement scores.

<sup>4</sup>Ranking the observations in descending order will produce the exact same statistic.

- (b) Sum the ranks separately for each of the  $k$  measurement periods (or factor levels) across subjects. *This operation will lead to  $k$  rank sums  $r_{+1}, \dots, r_{+k}$  for all  $k$  factor levels.*
- (c) The test statistic is calculated as follows:

$$F_r = \frac{12}{nk(k+1)} \sum_{j=1}^k [\bar{r}_{+j} - (k+1)/2]^2. \quad (2.3)$$

The law of probability of this test statistic is approximated by the chi-square distribution with  $k - 1$  degrees of freedom. This approximation is known to hold even for moderate values of  $n$ .

For the Friedman test, there is only one possible alternative hypothesis that is formulated as shown in 2.2. Assuming that  $\alpha$  is the test's significance level, and that  $c_\alpha$  is the  $100(1 - \alpha)^{th}$  percentile of the Chi-square distribution with  $k - 1$  degrees of freedom ( $c_\alpha$  is also known as the critical value), the decision rule is formulated as follows:

$$\text{Reject } H_0 \text{ if } F_{r(obs)} \geq c_\alpha \quad (2.4)$$

### Validity Conditions

The conditions that must be satisfied to ensure the validity of the Friedman test are very general and met most the times. Several authors such as Conover (1980, 1999), Daniel (1990) have listed the following validity conditions:

- The sample of subjects being analyzed was randomly selected from the population it represents. *In practice, the subjects are not always selected following a rigorous random selection protocol. The key aspect here is to have a sample that is a reasonably good representation of the population.*
- The analytic variable is a continuous variable. *Again, the Friedman test is often used successfully in practice with discrete variables that take numerous values.*

## 2.2 Tie Correction for the Friedman Test

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If your dataset contains ties, then it is widely-accepted in the statistical community that the Friedman test statistic must be divided by a tie-correction factor<sup>5</sup>  $C$  defined as follows:

$$C = 1 - \frac{\sum_{i=1}^T (t_i^3 - t_i)}{n(k^3 - k)}, \quad (2.5)$$

where  $T$  is the total number of tie series, and  $t_i$  is the number of tied scores in the  $i^{th}$  series of ties. Table 2.1 for example contains a single series of ties associated with salesperson 6. That series is  $\{1.5, 1.5\}$  and contains only 2 tied scores. Consequently,  $T = 1$ ,  $t_1 = 2$ . Since  $n = 9$ , and  $k = 4$ , the tie-correction factor  $C$  is given by  $C = (2^3 - 2)/(9(4^3 - 4)) = 0.98889$ .

Once the tie-correction factor  $C$  is calculated, it can be used to compute the tie-corrected Friedman test statistic as follows:

$$F_r^* = F_r / C. \quad (2.6)$$

Although tie correction is recommended primarily when the number of ties is excessive, it is nevertheless systematically implemented in several statistical packages.

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<sup>5</sup>The expression used for computing  $C$  (see equation 2.4) is based on a methodology described in Daniel (1990) and Marascuilo and McSweeney (1977)

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## 2.3 Pairwise Comparisons

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It follows from Figure 2.1 that the Friedman test led to the rejection of the null hypothesis based on Table 2.1 data. This suggests that the return rates is not uniform across days of the week, although there is no indication as to which days are different and which are similar. You may want to know whether the Wednesday return rate exceeds that of Tuesday for example. These are the follow-up analyzes that are also known as post-hoc analyzes.

To conduct the post-hoc analyzes, you as researcher needs to first determine the number of pairwise analyzes that you are interested in. In the clothing store return rate example, you may decide that you only want to know whether the Wednesday return rate exceeds that of Tuesday (i.e.  $H_a: \mu_{WE} > \mu_{TU}$ ). In this case you will have a single pairwise comparison of interest. If you want to compare Monday to Tuesday, Tuesday to Thursday, and Monday to Thursday then you will have 3 pairwise comparisons. Let me assume that you are interested in  $c$  pairwise comparisons. With  $c$  comparisons come  $c$  decisions to be made regarding the rejection or non-rejection of the different null hypotheses. In the context of pairwise comparison, you commit a type I error if any hypothesis in the family of  $c$  null hypotheses, is erroneously rejected. Your problem will then be to ensure that the probability of this special type I error does not exceed a specified *Familywise* significance level  $\alpha$ . This problem is resolved by setting a *Per Comparison* significance level of  $\alpha_{PC} = \alpha/c$  to be applied to each individual pairwise comparison before deciding whether the associated null hypothesis should be rejected or not.

If for example the Wednesday and Tuesday return rates must be compared, then you must first compute the Wednesday and Tuesday rank sums  $RS_{WE}$  and  $RS_{TU}$ . If you are testing equality of the 2 return rates (i.e. the null hypothesis of the 2-sided test), then you will reject it if the absolute difference  $|RS_{WE} - RS_{TU}|$  exceeds the threshold  $d_\alpha \sqrt{nk(k+1)/6}$ , where  $d_\alpha$  be the  $100(1 - \alpha_{PC/2})^{th}$  percentile of the standard Normal distribution. If on the other hand you are testing the hypothesis that the Tuesday return rate equals or exceeds that of Wednesday (i.e. the null hypothesis in the one-sided test), then you would reject it if the difference  $RS_{WE} - RS_{TU}$  exceeds  $d_\alpha \sqrt{nk(k+1)/6}$  where  $d_\alpha$  be the  $100(1 - \alpha_{PC})^{th}$  percentile of the standard Normal distribution.

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